Conic Sections: General Form of Conic Sections

A conic section can be written in the form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$,

Where A, B, C, D, E, and F are constants.

So if your equation was multiplied out like this and not in the pretty forms we were using:

Can you tell what type of conic section it is, just by the terms it has? Only the highest powered terms matter!

Case 1: $3x^2 - 12x - y + 7 = 0$

Only one of the variables is squared, so the conic section MUST be a parabola.

Rearranging the terms, you get $y = 3x^2 - 12x + 7$, which can be manipulated into the form:

$y = a(x-h)^2 + k$, so the parabola will open up or down.

The leading term is positive, so the parabola opens UP.

Case 2: $2x^2 + 2y^2 + 16x - 24y + 102 = 0$

The highest powered terms are $2x^2$ *and* $2y^2$, and both have THE SAME SIGN (in this case = positive)

Equations for circles, ellipses and hyperbola, do not have coefficients on leading terms.

So to remove the leading coefficients we can divide the entire equation by 2.

The result is: $x^2 + y^2 + 8x - 12y + 51 = 0$.

Completing the square (twice) yields an equation that

1. Has a "+" between the x^2 & y^2 terms and 2. DOESN'T have fractions. → So it's a circle!

$(x-h)^2 + (y-k)^2 = r^2$

Case 3: $16x^2 + 9y^2 - 96x - 36y - 396 = 0$

The highest powered terms are $16x^2$ *and* $9y^2$, and both terms have the SAME SIGN again.

To get rid of the leading coefficients, you'd have to divide the equation by 16 AND 9.

This leads to an equation of the form: $\frac{(x-h)^2}{9} + \frac{(y-k)^2}{16} = 1$ → So it's an ellipse (and it's vertical)

Case 4: $-4x^2 + y^2 + 4y - 12 = 0$

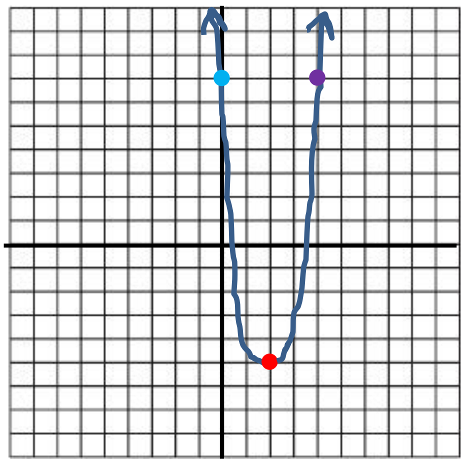
The highest powered terms are $16x^2$ *and* $9y^2$, and both terms have the OPPOSITE SIGNS.

To get rid of the leading coefficients, you'd have to divide the equation by 4 AND 1.

This leads to an equation of the form: $\frac{(y-k)^2}{4} + \frac{(x-h)^2}{1} = 1$ → So it's a hyperbola (and it's vertical)

Graph:

$y = 3(x-2)^2 - 5$



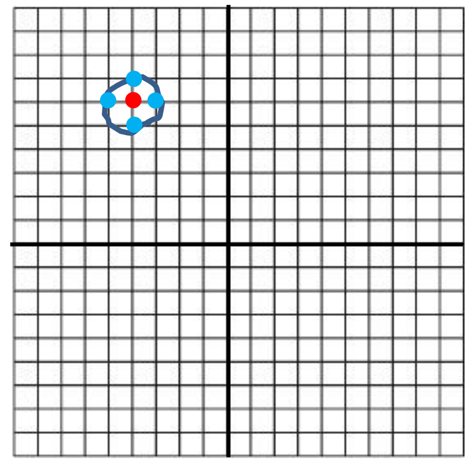
1. The center is (2, –5)
2.

x	y
0	7

Plot this point

3. Use symmetry for another point
4. Connect the dots

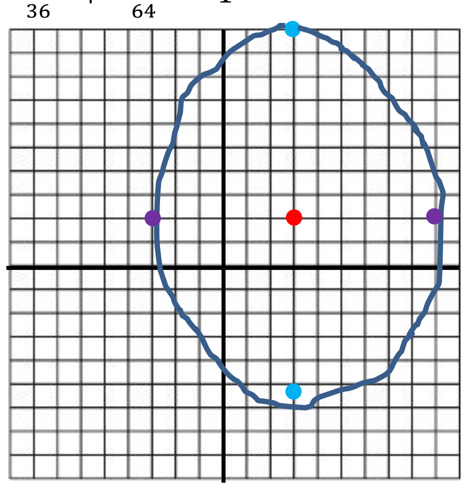
$(x+4)^2 + (y-6)^2 = 1$



1. The center is (–4, 6)
2. $r^2 = 1,$ so the radius = 1
3. Draw points 1 unit away from the center

   (left, right, up and down)
4. Connect the dots

$\frac{(x-3)^2}{36} + \frac{(y-2)^2}{64} = 1$

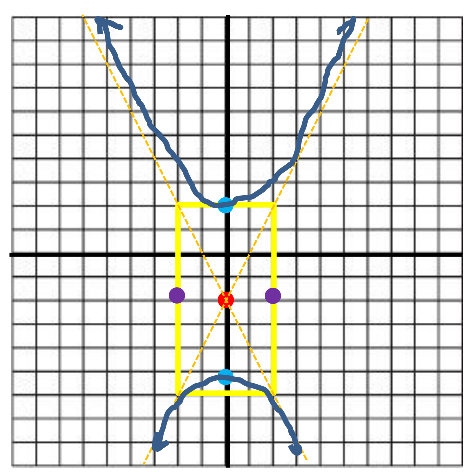


1. The center is (3, 2)
2. $a^2 = 64, a = 8$ (major axis is vertical)

   Draw a point up 8 and down 8.
3. $b^2 = 36, a = 6$ (minor axis is horizontal)

   Draw a point right 6 and left 6.
4. Connect the dots

$\frac{(y+2)^2}{16} - \frac{x^2}{4} = 1$



1. The center is (0, –2)
2. $a^2 = 16, a = 4$ (transverse axis is vertical)

   Draw a point 4 up and 4 down.
3. $b^2 = 4, a = 2$ (conjugate axis is horizontal)

   Draw a point right 2 and left 2.
4. Draw a box through the points
5. Draw asymptotes through corners of the box
6. Draw the halves of the hyperbola, going through the

   Points ( ) and using the asymptotes as guides.